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## A study of emergence in the generation of Islamic geometric patterns'

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## A STUDY OF EMERGENCE IN THE GENERATION OF ISLAMIC GEOMETRIC PATTERNS

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**Abstract.** Generative design is concerned with the definition and exploration of design spaces, and it has been suggested that emergence plays a key role in this process. In this paper, the impact of emergence on a design space is explored via consideration of different methods used to generate designs in a particular style. Three distinct methods of generating Islamic geometric patterns have been investigated and the extent to which emergence is employed in these methods has been explored. This research supports a discussion on the role of emergence in generative design, and an investigation into how design spaces are affected by the type of emergence employed in a generative process.

**Keywords.** Islamic geometric patterns; emergence; design space; design generation

### 1. Introduction

The definition and exploration of design spaces is a fundamental activity in generative design (Prats et al., 2006). As a result, the design space is a primary object for research intended to realise computational systems that support generative design (Woodbury and Burrow, 2006). This paper presents a discus-

sion on the relation between emergence in a generative system of design and the resulting design space. Intuitively, a relation does exist, since emergence in a generative process is likely to result in the generation of a broader range of designs, thereby expanding the boundaries of a design space. However, the extent to which this intuitive understanding is true remains to be seen.

Here, an exploration of this issue is presented which is based on consideration of different approaches to generating designs within the same style. Three different methods of generating Islamic geometric patterns are investigated, and because the different methods generate designs in the same style they provide a common ground from which to explore the relation between the use of emergence in a generative process, and the resulting design space. The aim of this exploration is to achieve a deeper understanding of the role of emergence in design generation and hence inform future development of computational systems intended to support generative design.

## **2. Islamic geometric patterns**

Throughout the world, the artistic traditions of Islam are apparent in a rich system of ornamentation applied to art and architecture. Islamic ornamentation consists of combinations of three predominant types of decoration – calligraphy, vegetal motifs and geometric patterns. Figural forms of humans or animals are not typical, in response to Qur’anic commandments against idolatry. Instead, abstract forms of decoration are used to create visual statements about religious ideas and express the logic and order inherent in the Islamic vision of the universe.

In this paper, methods for constructing Islamic geometric patterns are investigated as illustrations of generative methods of design. The patterns can be seen to be composed of simple polygons such as squares, triangles and stars, as illustrated in figure 1. However, from the combination and repetition of these polygons complex forms emerge which exhibit properties of symmetry, scalability and movement. As such, the patterns support exploration into philosophical aspects of Islamic faith, such as the relationship between unity and multiplicity.

In section 4 three methods for generating Islamic geometric patterns are presented. Two of these are based on historical techniques of construction, while the third is based on the shape grammar formalism (Stiny, 2006). The role of emergence in the three methods is discussed, and in order to support this discussion section 3 explores the concept of emergence in generative design. Finally, section 5 explores the relation between the role of emergence in a generative design process and the resulting design space.

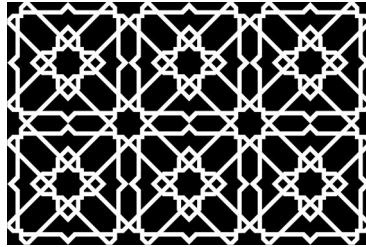


Figure 1. An Islamic geometric pattern.

### 3. Emergence in Islamic geometric patterns

On comparing the three methods of construction introduced in the next section, one differentiating factor that can be highlighted is the extent to which emergence plays a role. The concept of emergence is not always easy to define. For example, in complexity theory emergence “refers to the property of a collection of subunits that comes about through the interactions of the subunits and is not a property of any single subunit” (Flake, 1998, p. 449). With respect to geometric design, emergence can be defined as the construction of a shape that is not explicitly represented or identified in the method used to construct it. In this mode, it has been suggested that emergence plays a key role in design generation and exploration, as discussed by Schön and Wiggins (1992), Gross (2001) and Knight (2003). Consequently, it is of interest to support the recognition and use of emergence in computer based representations of shape. However, the problem of supporting emergence is exacerbated by the fact that it has many different guises.

Gross (2001) identified three types of emergence that should be supported in computer representations of shape. These are type 1) emergence that results from intersecting shapes, type 2) emergence that results from alternative configurations of parts of shapes, and type 3) emergence that results from figure-ground reversal. The methods for generating Islamic geometric patterns presented in the next section provide illustrations of all three of these types. However, before exploring these in detail it is beneficial to introduce a further distinction. This is the distinction between emergence as *process* and emergence as *product*, which Knight (2003) explores at length.

Emergence as process is characterised by emergent forms that are both recognised *and* used. For example, a foundational feature of the shape grammar formalism is that emergent shape is recognised and used in the process of generating designs (Stiny, 2006). Emergence as product on the other hand is characterised by emergent forms that are recognised but are *not* used. For example, cellular automata are often cited as examples of emergence since they are defined by simple rules but give rise to complex forms (Flake, 1998).

However, the emergence in a cellular automaton does not inform the process of defining the resultant form – it is emergence that is seen and not used, i.e. it is emergence as product.

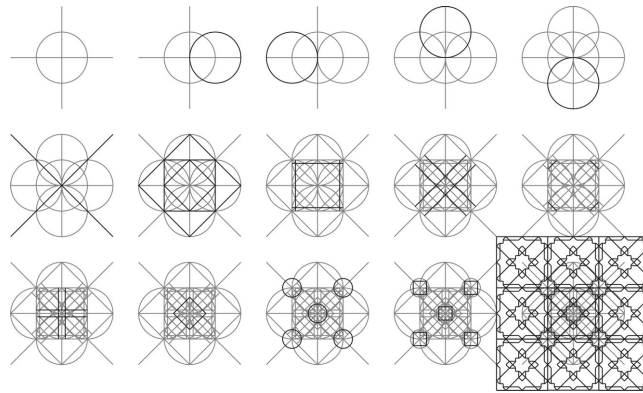
With respect to Islamic geometric patterns, both emergence as process and emergence as product are apparent. For example, a key characteristic of Islamic geometric patterns lies in the fact that they can be read in an unlimited number of ways. Ultimately, they can be seen to be composed of a set of simple polygons, but in the combination and repetition of these polygons there emerges an infinite range of complex geometric forms. This is illustrated in figure 1, where the combination of simple polygons results in the emergence of forms such as crosses and rosettes. Indeed, the range of forms that can be recognised in Islamic geometric patterns is limited only by the eye of the viewer. Some of these forms are intended by the designer, are included in the construction process, and result from emergence as process. The remaining forms are not included in the construction process and are emergence as product. In the next section three methods of generating Islamic geometric patterns are introduced, and for each method the type of emergence employed is identified by considering the underlying structure of generated patterns.

## **4. Generating Islamic geometric patterns**

### **4.1. COMPASS AND RULER CONSTRUCTION**

Very little information about the historical techniques for constructing Islamic geometric patterns survives to the present day. However, explorations into the geometrical properties of the patterns have resulted in a wide variety of different graphical techniques, including the use of set squares, grids and stencils, e.g., El-Said (1993). Fundamentally, each of these can be reduced to a method of construction using a compass and ruler. This method has its roots in the mathematics of ancient Greece, as recorded in Euclid's *Elements*, which was translated into Arabic by early Islamic scholars. In this ancient geometry, the straight-edge is an abstraction of a ruler – it being infinitely long, one-sided and having no markings. As a result it cannot be used for measurement but can be used to draw straight lines. Similarly, the abstraction of the compass can be opened arbitrarily wide, but when not in use it collapses. As a result it too cannot be used for measurement, but it can be used to draw circles. In practice the compass and ruler define operations that result in the creation of points, lines or circles. A point is created at the intersection of two lines, or at the intersection of two circles, or at the intersection of a line and a circle. A line is created by connecting two existing points, and a circle is created through one point with a second point as its centre.

In the construction of Islamic geometric patterns, these simple operations are applied repeatedly, as illustrated in figure 2. Here, in the first image, a simple initial construction is presented, composed of a circle and two lines. The intersections of the lines and circles define points which in turn are used to define new circles. Creation of the circles in turn results in the definition of new intersection points which are then used to define additional lines or circles. And the process can continue ad infinitum. As more lines and circles are added, a complex and intricate grid is defined. An Islamic geometric pattern is then derived from this grid by systematically selecting a set of line segments and circular arcs that are sub-sets of the grid lines.



*Figure 2. Compass and ruler construction of an Islamic geometric pattern.*

In compass and ruler construction the structure of a pattern is defined by the grid which results from the geometric operations of creating lines or circles. Emergence is fundamental in the process of defining this grid and in particular emergence type 1) – emergence that results from intersecting shapes – is predominant. As discussed, the grid is created according to points that emerge at the intersection of lines and circles. These emergent intersection points are then used to create new lines and circles. The final form of the grid is not predefined but has infinite variety that emerges as a result of the construction process. The final form of a geometric pattern is defined by systematically selecting emergent forms in the grid, such as lines or polygons. If the designer intends the pattern to be symmetric, then the construction will also incorporate emergence type 2) – emergence that results from alternative configurations of parts of shapes. This results from the designer seeing in part of the grid an emergent configuration that can be repeated, and then evolving the rest of the grid so that the configuration is repeated under symmetry transformations.

#### 4.2. SET-BASED CONSTRUCTION

The Islamic architecture of Spain and Morocco is distinctive in its ornamentation, since it makes use of monochrome mosaic tiles which are combined in a tessellation of the plane. This tile work, or *Zillij*, recreates Islamic geometric patterns by combining sets of small enamel hand-cut tiles called *furmah*. The *furmah* are defined according to the simple polygons that can be seen to compose the geometric patterns. For example, figure 3 illustrates a set of fifteen *furmah* that were identified in the analysis of a corpus of eight patterns, and named according to Hedgecoe and Damluji (1992). The variety of *furmah* used in *Zillij* far exceeds the examples illustrated here. For example, it is estimated there are 360 *furmah* in common use, but uncountable more are apparent in historical examples of *Zillij*. While the great masters of *Zillij* continue to practice their trade, the secret to their art remains a mystery. No instructive or analytical writings about the art are known of, and instead the secret is passed on only from master to apprentice (Hedgecoe and Damluji, 1992). However, the mysteries can be discovered, to a certain extent, via geometric study of *Zillij* themselves.

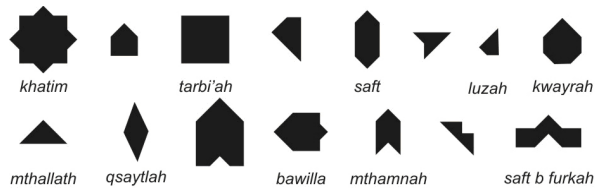


Figure 3. A set of fifteen *furmah*.

For the study presented in this paper, a method of constructing Islamic geometric patterns was defined based on *Zillij*. In the method geometric patterns are generated according to the set of fifteen *furmah* illustrated in figure 3. This choice of *furmah* is not unique but is a consequence of the eight tiles in the analysed corpus – analysis of an alternative corpus would likely lead to the selection of an alternative set of *furmah*. In generating a pattern, the *furmah* can be transformed according to similarity transformations – which are combinations of translation, rotation, reflection and isotropic scaling. However, a legal pattern can contain no holes, nor can it include parts of *furmah*. For example, figure 4 illustrates three designs generated using this set-based construction method.

In set-based construction the structure of a pattern evolves with the pattern itself – as new *furmah* are added to a design the structure grows accordingly. The structure is an emergent product of the construction process. As with compass and ruler construction this structure has an infinite variety, but here

it is limited according to the set of available *furmah*. In considering set-based construction two different types of emergence can be identified; neither is necessary in the construction of a pattern, and one is illegal. Firstly, type 2) emergence can be utilised in order to incorporate symmetry in a design, in a manner similar to that described for compass and ruler construction. Secondly, type 3) emergence – emergence that results from figure–ground reversal – can be identified in the recognition of holes that result from piecing *furmah* together. It is likely that figure–ground reversal is the reason for the uncountable number of *furmah* that are apparent in historical examples of *Zillij*. However, in the tiling–based construction this emergence is not allowed since it was specified that patterns must not contain any holes and they must be composed only of the *furmah* defined in figure 3.

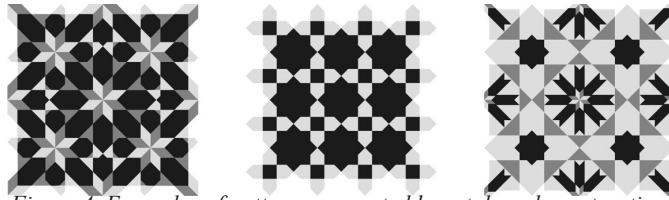


Figure 4. Examples of patterns generated by set-based construction.

#### 4.3. MOTIF-BASED CONSTRUCTION

The geometry of Islamic patterns can also be rigorously described according to the shape grammar formalism (Stiny, 2006). A shape grammar consists of shape rules that define geometric properties of a particular style. The rules can then be applied to generate existing or new instances of the style.

For the purposes of this study, a corpus of eight Islamic geometric patterns was analysed (this was the same corpus used in defining the method of set-based construction). Each of the patterns in the corpus exhibited a repeating square pattern with an eight-fold rotational symmetry, and the underlying structure could be described according to a triangular region, the contents of which serve as the motif of the pattern. The resultant shape grammar, illustrated in figure 5, uses the symmetry of the patterns in order to remove any redundancy in the generative process, according to two types of rule. The first type, rules 1–8, is concerned with defining the geometric properties of the triangular motif. The second type, rules 9 and 10, is concerned with repeating the motif under symmetry transformations.

Application of a shape grammar is initialised by a seed shape, which for the Islamic geometric pattern grammar is a right-angled triangle, as illustrated in figure 6. Here, the step-by-step generation of an Islamic geometric pattern is presented, and the rules that are applied to move from one design to the next



are specified. Rule 1 adds a grid to the initial triangle, and rules 2–6 define line elements of the motif by recognising and replacing grid lines. The grid and the lines added by rules 2–6 were derived from the analysis of the eight patterns in the corpus. As a result, they cannot be claimed to universal since analysis of an alternative corpus would likely lead to the definition of an alternative rules. When the designer is satisfied with the generated motif he can prevent any further modification via application of rules 7 and 8, which remove grid lines and their associated end points.

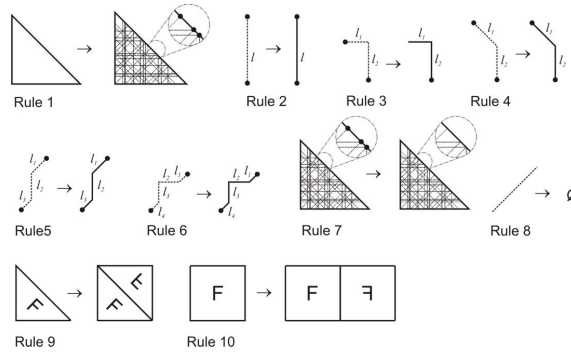


Figure 5. Shape rules of the Islamic geometric pattern grammar.

The last two rules in the grammar, rules 9 and 10, define how the generated motif is repeated under symmetry transformations. In these rules the *F* denotes any generated motif. Knight (2003) defines rules such as these as *emergence rules*. The rules are applied to an emergent design that has resulted from previous rule applications. Rule 9 recognises the triangular motif (including the pattern lines added by previous rules applications) and reflects this through the triangle's hypotenuse. The result is a square containing a pattern with a reflective symmetry. Rule 10 recognises the square pattern that results from the application of rule 9 and reflects this through an edge of the square. Repeated application of rule 10 results in a larger square pattern, with an eight-fold rotational symmetry.

Motif-based construction can be seen as a two stage process, where in the first stage the motif is defined, and in the second it is repeated under symmetry transformations. Definition of the motif does not incorporate any emergence since it is based on a fixed grid. However, the motif is an emergent product of this stage in the process, and according to type 2) emergence it is recognised and used in the second stage of the process.

## 5. Generating design spaces

Section 3 explored the concept of emergence in generative design and, based on previous research, three types were identified. In addition, a further distinction was made between emergence as process (where emergent forms are both recognised *and* used) and emergence as product (where emergent forms are recognised but *not* used). In generative design, emphasis is often placed on supporting emergence as process, e.g. it is a foundational feature of the shape grammar formalism (Stiny, 2006). However, emergence as product can also play a role in the generative process. For example, the discussion in section 4 revealed that emergence as product plays a role in the process of generating Islamic geometric patterns in two distinct ways.

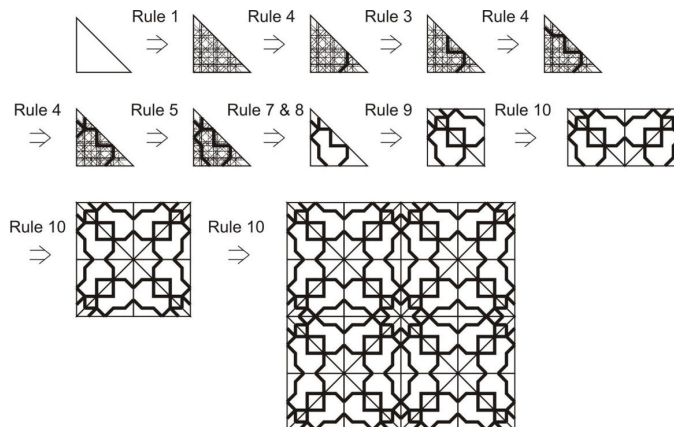


Figure 6. Motif-based construction of an Islamic geometric pattern.

Firstly, emergence as product can play a role when a generative process is a multi-stage process. For example, this was illustrated in motif-based construction (section 4.3), where the first stage of the process generates a motif as an emergent product; the second stage recognises this and uses it in the process of generating a pattern. Similarly, compass and ruler construction (section 4.1) can also be viewed as a two-stage process. Here, the first stage results in definition of a grid as an emergent product, from which design elements are selected in the second stage. The idea of emergence as product being used this way in design generation is not new and has been explored previously, for example in Knight (2003).

Secondly, emergence as product can play a role when a designer fixates on a particular aspect of a design. For example, this was illustrated in set-based construction (section 4.2) where, if a designer intends a generated design to be symmetric, a particular configuration of the parts in a design is recognised as

an emergent product. This product is then repeated under symmetry transformations to create a pattern. A similar method is also used to impose symmetry on patterns constructed using compass and ruler.

Interestingly, emergence as process, emergence as product, and emergence as product *in process* all have different properties with respect to the design space defined by a generative process. As mentioned, emergence as process is often emphasised in generative design, and this is because it can result in an expansion of the design space through the use of emergent forms. Emergence as product on the other hand is the output of generation and as a result has no effect on the design space. Emergence as product in process can have one of two different effects on the design space. Firstly, it can allow the design space to be populated, by enabling the design process to continue. This corresponds to emergence as product in a multi-stage generative process. Secondly, it can result in the shrinking of the design space. This corresponds to fixation on particular aspects of a design, resulting in a disregard for the alternatives.

The aim of this discussion was to inform future development of computational systems intended to support generative design, and to this end it has highlighted some of the complications that can arise when considering the concept of emergence in design generation. In particular, different types of emergence all have a significantly different impact on the design space defined by a generative process. Despite this, processes that incorporate different types of emergence can be used to generate designs within the same style, as illustrated in section 4. This suggests that the extent to which emergence needs to be supported in a system depends on the extent to which the defined design space will be explored. This merits further investigation.

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